Bracket Notations Part-I Arvind W Kiwelekar

Bracket Notations

- 1. Also Known as Dirac Notation because Physicist Paul Dirac has introduced it.
- 2. It is a compact and convenient way of representing quantum states and operations in quantum computing.
- 3. It is widely used because it simplifies complex mathematics of quantum theory.

 $\langle \textit{Bra} | \textit{ket} \rangle$

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Different Forms of Bracket Notations

| Ar. No. | Name | Symbol | Purpose |
|---------|-------------------|---------------------------|--------------------|
| 1 | Ket Notations | $.\rangle$ | Column Vector |
| 2 | Bra Notations | ⟨. | Row Vector |
| 3 | Ket-Ket Notations | $.\rangle .\rangle$ | Tensor Product of |
| | | | Column Vector |
| 4 | Bra-Bra Notations | $\langle . \langle . $ | Tensor Product of |
| | | | Row Vector |
| 5 | Ket-Bra Notations | $.\rangle\langle. $ | Matrix Representa- |
| | | | tion |
| 6 | Bra-Ket Notations | $\langle . . \rangle$ | Inner Product |

(1) Ket Notations

Representing Quantum States through Column Vectors

$$|0
angle = egin{pmatrix} 1 \ 0 \end{pmatrix} \qquad |1
angle = egin{pmatrix} 0 \ 1 \end{pmatrix}$$

Converting Ket to Column Vectors

 $|11
angle = egin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

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 $D = 2^n$ dimensional column vector represents *n* bit information.

(1) Ket Notations

Representing Column Vectors through Ket notations

Example 2

Example 1

$$egin{pmatrix} 7 \ 3+5i \end{pmatrix} = 7 |0
angle + (3+5i) |1
angle$$

The basis used is $\left\{ \left| 0 \right\rangle, \left| 1 \right\rangle \right\}$

$$egin{pmatrix} 7 \ 0 \ i+3 \ 0 \end{pmatrix} = 7 |00
angle + (i+3) |10
angle$$

 $\begin{array}{lll} \text{The} & \text{basis} & \text{used} & \text{is} \\ \left\{ \left| 00 \right\rangle, \left| 01 \right\rangle, \left| 10 \right\rangle, \left| 11 \right\rangle \right\} \end{array}$

(2) Bra Notations

Representing Row Vectors through Bra notations

The ket and bra notations are interconvertible.

$$\begin{aligned} |\psi\rangle^+ &= \langle \psi| \\ \langle \psi|^+ &= |\psi\rangle \end{aligned}$$

Example in bra notations $\langle \psi | = (3 - 5i) \langle 0 | + 7 \langle 1 |$ $(3 - 5i \quad 7)$ Example in bra notations $|\psi\rangle = (3+5i) |0\rangle + 7 |1\rangle$ $\begin{pmatrix} 3+5i \\ 7 \end{pmatrix}$

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Representing Row Vectors through Bra notations

Example 2

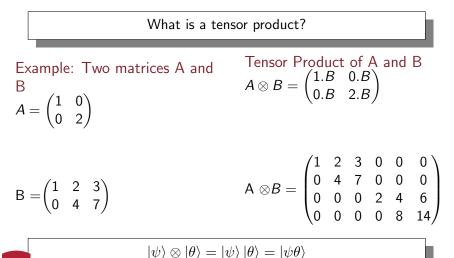
Here, the basis is $\left\{ \left<00\right|, \left<01\right|, \left<10\right|, \left<11\right| \right\}$

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 $\begin{pmatrix} 3 & 0 & i & 7 \end{pmatrix} = 3\langle 00| + i \langle 10| + 7 \langle 11| \end{pmatrix}$

(3) Ket-Ket Notations

It represents the tensor product of Column Vectors



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(3) Ket-Ket Notations

Example

Two Given Functions $\psi = i |0\rangle + 7 |1\rangle$

$$\phi = \ket{00} + 3 \ket{10} + 7 \ket{11}$$

Vector Representations $|\psi\rangle = \binom{i}{7}$

Tensor product:
$$|\psi\phi\rangle$$

$$ert \psi \phi
angle = i ert 000
angle + 3i ert 010
angle + 7i ert 011
angle + 7ert 100
angle + 21 ert 110
angle + 49 ert 111
angle$$

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Tensor product:
$$|\psi\phi\rangle$$

 $|\psi\phi\rangle = \begin{pmatrix} i.\phi\\ 7.\phi \end{pmatrix} = \begin{pmatrix} i\\ 0\\ 3i\\ 7i\\ 7\\ 0\\ 21\\ 49 \end{pmatrix}$