Bracket Notations
Part-II

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(4) Bra-Bra Notations

It represents the tensor product of row vectors.

Example

Two Given Functions

$\langle\psi| = 3\langle 0| + 7\langle 1|$

$\langle\phi| = \langle 0| + i\langle 1|$

Vector Representations

$\langle\psi| = \begin{pmatrix} 3 & 7 \end{pmatrix}$

$\langle\phi| = \begin{pmatrix} 1 & i \end{pmatrix}$

Tensor product: $\langle\psi\phi|$

$= \langle\psi|\langle\phi| =$
$3\langle 00| + 3i\langle 01| + 7\langle 10| + 7i\langle 11|$

Tensor product: $\langle\psi\phi|$

$\langle\psi\phi| = \begin{pmatrix} 3.\phi & 7.\phi \end{pmatrix} =$
$\begin{pmatrix} 3 & 3i & 7 & 7i \end{pmatrix}$

(5) Ket-Bra Notations

It represents a matrix.

$$|\psi\rangle \langle\phi| = |\psi_{m \times 1} X \phi_{1 \times n}|$$

First Example $|00\rangle \langle 10|$

Two Given Functions

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{pmatrix}$$

(5) Ket-Bra Notations (Second Example)

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$$|\psi\rangle = 3\,|0\rangle + i\,|1\rangle$$
$$|\phi\rangle = |00\rangle + 2\,|10\rangle + 7\,|11\rangle$$
$$\langle\phi| = |\phi\rangle^{+} = \langle 00| + 2\,\langle 10| + 7\,\langle 11|$$

Solution

$$|\psi X \phi| =$$
$$3|0X00| + 6|0X10| + 21|0X\ 11| + i|1X00| + 2i|1X10| + 7i|1X11|$$

Matrix Representations

No of rows $= 2^1 = 2$ No of columns $= 2^2 = 4$

$$\begin{pmatrix} 3 & 0 & 6 & 21 \\ i & 0 & 2i & 7i \end{pmatrix}$$

(5)Representing a matrix using Ket-Bra Notations

Example 1

$$A = \begin{pmatrix} 0 & 1 \\ 3 & i \\ 7 & 0 \\ 0 & 13 \end{pmatrix}$$

Solution

No of rows $= 4 = 2^2$. Hence Basis will be $\{0,01,10,11\}$
No of Columns $= 2 = 2^1$. Hence Basis will be $\{0,1\}$

$$A = |00X1| + 3|01X0| + i|01X1| + 7|10X0| + 13|11X1|$$

(6) Bra-Ket Notations: Representing Inner product

$\langle \alpha_{1xn}| \, |\beta_{mx1}\rangle$ when n=m the resultant vector is 1x1 means a unit vector or constant

Examples

$\langle 0| \, |0\rangle$

$=\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$= 1x1 + 0x0$

$=1$

$\langle 1| \, |1\rangle$

$=\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= 0x0 + 1x1$

$=1$

$\langle 0| \, |1\rangle$

$=\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= 1x0 + 0x1$

$=0$

$\langle 1| \, |0\rangle$

$=\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$= 0x1 + 1x0$

$=0$

(6) Bra-Ket Notations -Example

Problem

$|\psi\rangle = i\,|0\rangle + 7\,|1\rangle$

$|\phi\rangle = 3\,|0\rangle + |1\rangle$

Solution

$\langle\psi| = |\psi\rangle^{+} = -i\,\langle 0| + 7\,\langle 1|$

$\langle\psi|\,|\phi\rangle = (-i\,\langle 0| + 7\,\langle 1|)\,(3\,|0\rangle + |1\rangle)$

$=$ -3i$\langle 0|0\rangle - i\,\langle 0|1\rangle + 21\,\langle 1|0\rangle + 7\,\langle 1|1\rangle$

$=$ -3i $+7$

> Inner products are non-commutative
> for real: $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle$
> for complex: $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^{+}$