Bracket Notations Part-II Arvind W Kiwelekar

▲□▶ ▲□▶ ▲ 臣▶ ▲ 臣▶ ― 臣 … のへぐ

# (4) Bra-Bra Notations

It represents the tensor product of row vectors. Example

Two Given Functions  $\langle \psi | = 3 \langle 0 | + 7 \langle 1 |$ 

 $\langle \phi | = \langle 0 | + i \langle 1 |$ 

Vector Representations  $\langle \psi | = \begin{pmatrix} 3 & 7 \end{pmatrix}$ 

 $\langle \phi | = \begin{pmatrix} 1 & i \end{pmatrix}$ 

Tensor product:  $\langle \psi \phi |$ =  $\langle \psi | \langle \phi | =$ 3  $\langle 00 | + 3i \langle 01 | + 7 \langle 10 | + 7i \langle 11 |$ 

Tensor product:  $\langle \psi \phi |$  $\langle \psi \phi | = (3.\phi \quad 7.\phi) =$  $(3 \quad 3i \quad 7 \quad 7i)$ 

# (5) Ket-Bra Notations

It represents a matrix.

▲ロト ▲御 ト ▲ 臣 ト ▲ 臣 ト 二 臣 … のへで

Two Given Functions  $\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix}
(0 \quad 0 \quad 1 \quad 0)$ 

## (5) Ket-Bra Notations (Second Example)

$$\begin{split} |\psi\rangle &= 3 \left|0\right\rangle + i \left|1\right\rangle \\ |\phi\rangle &= |00\rangle + 2 \left|10\right\rangle + 7 \left|11\right\rangle \\ \langle\phi| &= |\phi\rangle^{+} = \langle00| + 2 \left\langle10\right| + 7 \left\langle11\right| \end{split}$$

### Solution $|\psi X \phi| =$ 3|0X00| + 6|0X10| + 21|0X 11| + i|1X00| + 2i|1X10| + 7i|1X11|

Matrix Representations

No of rows = 
$$2^1 = 2$$
 No of columns =  $2^2 = 4$   
 $\begin{pmatrix} 3 & 0 & 6 & 21 \\ i & 0 & 2i & 7i \end{pmatrix}$ 

### (5)Representing a matrix using Ket-Bra Notations

Example 1

$$A = \begin{pmatrix} 0 & 1 \\ 3 & i \\ 7 & 0 \\ 0 & 13 \end{pmatrix}$$

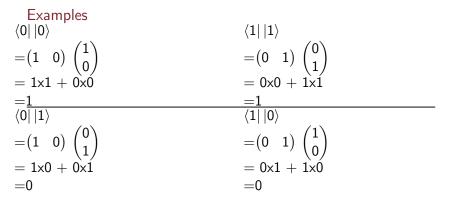
#### Solution

No of rows = 4 =  $2^2$ . Hence Basis will be {0,01,10,11 } No of Columns = 2 =  $2^1$ . Hence Basis will be {0,1 }

A = |00X1| + 3|01X0| + i|01X1| + 7|10X0| + 13|11X1|

(6) Bra-Ket Notations: Representing Inner product

 $\langle \alpha_{1\times n} | | \beta_{m\times 1} \rangle$  when n=m the resultant vector is 1×1 means a unit vector or constant



(6) Bra-Ket Notations -Example

 $\begin{aligned} & \text{Problem} \\ & |\psi\rangle = i \, |0\rangle + 7 \, |1\rangle \\ & |\phi\rangle = 3 \, |0\rangle + |1\rangle \end{aligned}$ 

Solution  

$$\begin{aligned} \langle \psi | &= |\psi \rangle^+ = -i \langle 0 | + 7 \langle 1 | \\ \langle \psi | |\phi \rangle &= (-i \langle 0 | + 7 \langle 1 |) (3 | 0 \rangle + | 1 \rangle) \\ &= -3i \langle 0 | 0 \rangle - i \langle 0 | 1 \rangle + 21 \langle 1 | 0 \rangle + 7 \langle 1 | 1 \rangle \\ &= -3i + 7 \end{aligned}$$

Inner products are non-commutative for real:  $\langle \alpha | \beta \rangle = \langle \beta | \alpha \rangle$ for complex:  $\langle \alpha | \beta \rangle = \langle \beta | \alpha \rangle^+$ 

24