Two Input Quantum Controlled Gates
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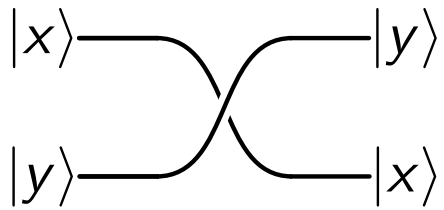
Two Input Quantum Gates: Swap Gate

- All two input quantum gates are 4 X 4 matrices.

The symbol is



The matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

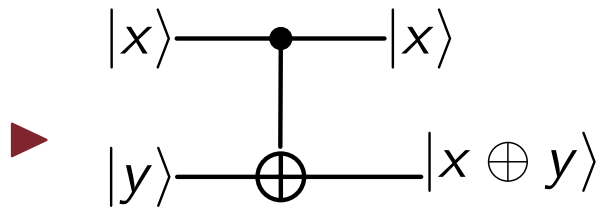
- The working is-
  1. $swap\,|00\rangle = |00\rangle$
  2. $swap\,|01\rangle = |10\rangle$
  3. $swap\,|10\rangle = |01\rangle$
  4. $swap\,|11\rangle = |11\rangle$

$$swap\,|01\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = |10\rangle$$

Two Input Quantum Controlled Gates: CNOT Gate

- All two input quantum gates are 4 X 4 matrices.

The symbol is



The matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The first input is controlled.

- The working is-
  1. $swap\,|00\rangle = |00\rangle$
  2. $swap\,|01\rangle = |01\rangle$
  3. $swap\,|10\rangle = |11\rangle$
  4. $swap\,|11\rangle = |10\rangle$

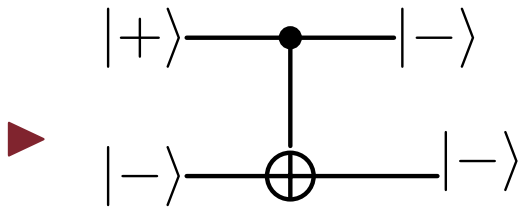
$$CNOT\,|10\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle$$

Working of CNOT Gate on Superimposed Gate

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle$$

The symbol is



The working is

1. $CNOT|{+}{+}\rangle = |{+}{+}\rangle$
2. $CNOT|{-}{+}\rangle = |{-}{+}\rangle$
3. $CNOT|{-}{-}\rangle = |{+}{-}\rangle$
4. $CNOT|{+}{-}\rangle = |{-}{-}\rangle$

The Second input is controlled.

$$|{-}{-}\rangle = |-\rangle|-\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}} \cdot \frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{|00\rangle-|01\rangle-|10\rangle+|11\rangle}{2} = \frac{1}{2}\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}$$

$$|{+}{-}\rangle = |+\rangle|-\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}} \cdot \frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{|00\rangle-|01\rangle+|10\rangle-|11\rangle}{2} = \frac{1}{2}\begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}$$

Working of CNOT Gate on Superimposed Gate

To verify $CNOT\,|{-}{-}\rangle = |{+}{-}\rangle$ $CNOT\,|{-}{-}\rangle$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1+0+0+0 \\ 0-1+0+0 \\ 0+0+0+1 \\ 0+0-1+0 \end{pmatrix}$$

$$\frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$= |{+}{-}\rangle$$

General CU gate Single Input Qubit

General U gate

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u & u \\ 0 & 0 & u & u \end{pmatrix}$$

Controlled Z gate (CZ)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

Controlled H gate (CH)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Resources

- Introduction to classical and Quantum Computing by Thomas Wong